96.9.

歸檔

國立中央大學數學系九十六學年度博士班資格考試題卷

科目：常微分方程

1.(10%) Let r, k, and f be real and continuous functions defined in $[a, b]$, $k(t) \geq 0$ for $t \in [a, b]$, and

$$r(t) \leq f(t) + \int_a^t k(s)r(s)ds, \ a \leq t \leq b.$$

If $f(t)$ is differentiable, show that

$$r(t) \leq f(a)(\int_a^t k(u)du) + \int_a^t \exp(\int_s^t k(u)du)f'(s)ds, \ a \leq t \leq b.$$

2.(20%) Assume $k > 0$, $g(t)$ is continuous on (a, b) and ϕ be a nontrivial solution of $ky'' + g(t)y = 0$.

(1). If ϕ has at least $(n+1)$ zeros in (a, b), show that

$$\sup\{g(t) : \ a < t < b\} > (\frac{n\pi}{b-a})^2 k.$$

(2). If $\inf\{g(t) : \ a < t < b\} > (\frac{n\pi}{b-a})^2 k$, show that there is a nontrivial solution of $ky'' + g(t)y = 0$ which has at least n zeros.

3.(10%) Determine the system

$$x' = -4y + x(1 - \frac{x^2}{4} - y^2),$$
$$y' = x + y(1 - \frac{x^2}{4} - y^2).$$

has periodic orbits or not. If there are periodic orbits, please determine their stability.

4. (20%) Consider the following system:

$$x' = x - y - x^3 \quad y' = x + y - y^3.$$

(1) Show that there is a unique equilibrium.

(2) Show that there is a unique stable limit cycle in the region

$$A = \{x \in R^2 | \ 1 \leq |x| \leq \sqrt{2} \ \}.$$

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5. (20%) Let $a(t)$ and $b(t)$ be continuous and T-periodic functions, ϕ_1 and ϕ_2 be solutions of $y'' + b(t)y' + a(t)y = 0$ such that

$$\phi_1(0) = 1, \phi_2(0) = 0, \phi_1'(0) = 0, \phi_2'(0) = 1.$$

Find the Floquet multipliers of the equation.

6. (10%) Consider the second order system:

$$x'' + x' + e^{-t}x = 0.$$

Determine the stability of the trivial solution $x \equiv 0$.

7. (10%) Find the general solutions of the following system:

$$\begin{aligned} \frac{dx}{dt} &= 3x + y - z \\ \frac{dy}{dt} &= x + 3y - z \\ \frac{dz}{dt} &= 3x + 3y - z \end{aligned}$$