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DEPARTMENT OF MATHEMATICS
NATIONAL CENTRAL UNIVERSITY

Ph. D. Qualifying Examination

Fall, 2002

Analysis

1. Suppose $\int_I f(y)dy = 0$ for every subinterval I of $\mathbb{R}$. Show: $f = 0$ a.e. on $\mathbb{R}$.

2. If $f \in L^1(-\infty, \infty)$ then show:
$$\lim_{n\to\infty} \int_{-\infty}^{\infty} f(x) \cos\, nx\, dx = 0$$

3. Let $F(x, y)$ be continuous and bounded on the unit square $0 < x < 1$, $0 < y < 1$. Show: $f(y) = \overline{\lim}_{x\to 0+} F(x, y)$ is measurable on $(0, 1)$.

4. Suppose $f,\ f_n$ are of bounded variation on $[a, b]$, $n = 1, 2, \cdots$ and
$$V(f_n - f, a, b) \to 0$$
(where $V(f, a, b) =$ the total variation of f on $[a, b]$)
Prove: There is a subsequence $n_k \to \infty$ such that
$$f'_{n_k} \longrightarrow f' \quad \text{a.e.}$$

5. Let $f \in L(\mathbb{R}^n)$. Show
$$\lim_{|h|\to 0} \int_{\mathbb{R}^n} |f(x + h) - f(x)|dx = 0$$

6. Let $f, g \in L^1(\mathbb{R}^1)$. Show:
   (a) $f(x - t)g(t)$ is a measurable function on $\mathbb{R}^2$.
   (b) $\phi(x) = \int_{-\infty}^{\infty} f(x - t)g(t)dt$ is in $L^1(\mathbb{R}^1)$

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7. Let $(X, \mathfrak{B})$ be a mesurable space and on it, ν is a finite measure and μ a σ-finite measure, show the following are equivalent:
   (a) $\nu \ll \mu$ and
   (b) If the sequence $E_n \subseteq \mathfrak{B}$ has the property that $\lim_{n\to\infty} \mu(E_n) = 0$, then $\lim_{n\to\infty} \nu(E_n) = 0$.

8. (a) Show: The normed vector space $L^p(\mu)$ is a Banach space.
   (b) Let ϕ be a continuous linear map from $\mathbb{R}^n$ into a normed vector space F. Find the sup norm of ϕ.