中央大學數學系博士班入學考試 2009.05.15

歸檔

圖論 Graph Theory

1. Show that every graph G contains a bipartite spanning subgraph H such that $\deg_H(v) \geqq 1/2 \deg_G(v)$ for all v in V(G). (15 分)

2. Let X be a set with n elements, and $A_1, A_2, \ldots\ldots, A_n$ be distinct subsets of X. Show that there exists an element x in X such that $A_1-\{x\}, A_2-\{x\}, \ldots\ldots, A_n-\{x\}$ are distinct. (20 分)

3. Show that the complete graph K_n ($n \geqq 2$) has edge chromatic number n-1 if n is even and n if n is odd. (15 分)

4. Show that for any positive integer k, these exists a graph G with chromatic number k such that G contains no triangle. (15 分)

5. Let G be a connected graph of order n and let m be a positive integer $\leqq$ n-1. Suppose that deg x+deg y $\geqq$ m for every nonadjacent vertices x , y in G. Show that G contains a path of order m. (15 分)

6. Let n , k be positive integers such that $k \geqq 2$ and $n < 2^{k/2}$. Show that there exists a graph G of order n such that G contains neither a clique of k vertices nor an independent set of k vertices. (20 分)