歸檔

國立中央大學數學系 96 學年度博士班入學考試筆試試題

科目：代數

博士班入學考 (Algebra 2007)

1. (a) Describe what is the Zorn's Lemma? (5%)

   (b) Describe without proof the Fundamental Theorem of Galois Theory? (5%)

2. Let G be an abelian group and I, J be any infinite subgroups of G with G/I and G/J being finite groups. Prove or disprove that $I \cap J$ is an infinite subgroup of G. (15%)

3. What is the Galois group of the field $\boldsymbol{R}$ of real numbers over the field $\boldsymbol{Q}$ of rational numbers? Verify your assertion. (15%)

4. Give a noncommutative ring with unique maximal ideal. Verify your assertion. (20%)

5. Prove that the field $\boldsymbol{Q}$ of rational numbers is not a projective $\boldsymbol{Z}$-module, where $\boldsymbol{Z}$ denotes the ring of rational integers. (20%)

6. Let P and Q be distinct prime ideals in a Dedekind domain D. For any positive rational integers a and b, prove that $P^a + Q^b = D$. (20%)